THE METHOD OF LINES FOR THE HYBRID ANALYSIS OF MULTILAYERED DIELECTRIC RESONATORS

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Abstract

In this paper the Method of Lines has been used in the hybrid analysis of dielectric resonators. The resonant frequencies, as well as Q- factors caused by radiation or dielectric loss and the corresponding field distributions, will be determined with the described algorithm based on the Method of Lines. For the five lowest order modes the resonant frequencies and the Q-factors due to radiation are presented for a dielectric resonator of the relative permittivity of 38. The results are compared with those of other rigorous numerical methods and with the newest measurement available in the literature.

1 Introduction

Dielectric resonators are utilized in circuits in integrated microwave engineering for the stabilization of oscillators or as filter elements. The increasing importance of mobile telecommunication systems and the shift from vehicle mounted mobile phone systems to portable units has increased the demand for dielectric resonators with a small size, low cost and good temperature stability. Because of the proximity of the resonant frequencies of the various hybrid modes to one another, it is very important to know all the exact resonant frequencies and the corresponding field distributions in the frequency range of interest. Currently there are several approaches available for the rigorous analysis of cylindrical dielectric resonators [1]. Most of them are only useful for the determination of resonator modes without azimuthal variation or are restricted to particular geometries. The method of moments based on the surface integral equation [2],[3], the null-field method [4] and a combination of the finite-difference time domain and Prony's method [5], however, have been further developed to enable the hybrid analysis of isolated dielectric resonators.

It has been shown in a number of papers that the method of lines (MoL) [6] is highly suitable for the analysis of electromagnetic field problems. The introduction of absorbing boundary conditions in the MoL has made it possible to simulate radiating structures such as microstrip patch antennas [7][8] and cylindrical antennas [9].

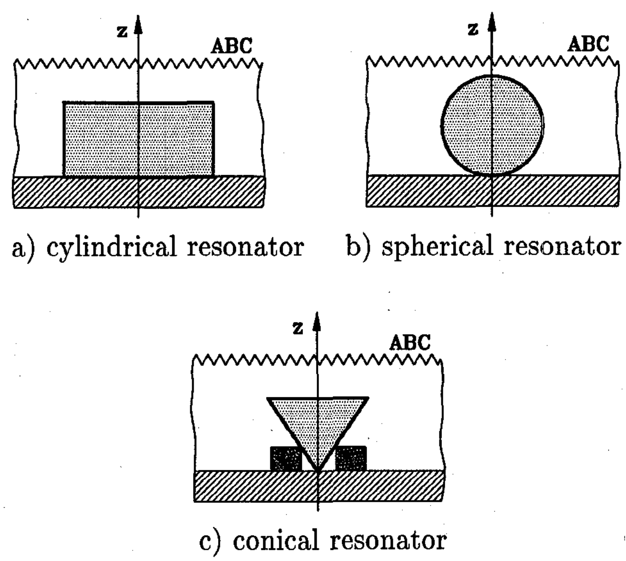

Fig. 1: Resonator structures

The purpose of this paper is to show that the MoL can be applied very efficiently to the analy-

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sis of cylindrical dielectric resonators. Our model enables us to investigate resonator structures with different shapes, e. g. rotationally symmetric, spherical or conical forms (see Fig. 1) placed on a substrate. As a surrounding boundary of the structure we can use either absorbing boundary conditions (ABC), or metallic or magnetic walls. The resonator structure is modeled by a set of single coaxial cylindrical rings in the radial direction. These rings show permittivities that depend on the z direction. Lossy materials can be considered using complex permittivities. Such multilayered dielectric resonators surrounded by metallic walls were first investigated by [10] using an algorithm based on the mode matching technique and [11] with an differential method.

2 Theorie

In cylindrical coordinates the electromagnetic field for an inhomogeneous dielectric medium can be determined by two vector potentials $\mathbf{\Pi}_e$ and $\mathbf{\Pi}_h$, which have only one component ψ_e and ψ_h in the z direction, respectively. If we assume symmetrical resonator structures relating to $z = 0$, a harmonic time-dependence in accordance with $e^{j\omega t}$ and a space dependent on the permittivity according to $\varepsilon_r(z)$, the scalar potentials ψ_e and ψ_h must fulfill the Sturm-Liouville differential equation and the Helmholtz equation in cylindrical coordinates, respectively. For solution of these differential equations the potentials and

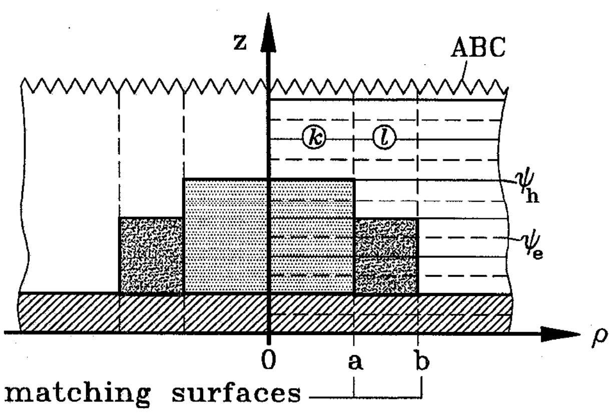

Fig. 2: Discretized multilayered resonator structure

the permittivities ε are partly discretized perpendicular to the axis of rotation. Fig. 2 shows the longitudinal section of the discretized computational window. Because the discretization is only enforced in the z direction, the potentials and fields are analytically calculated on lines in the radial direction. We use two different line systems for the potentials ψ_e and ψ_h that are shifted towards each other to satisfy the interface conditions in discretization direction.

Substituting the corresponding difference operators in place of the differential operators in the wave equations yields two sets of coupled differential equations. After decoupling these equations by transforming them to the main axis, we obtain a system of uncoupled differential equations for the transformed potentials $\tilde{\boldsymbol{\psi}}_{e,h}$

$$\overline{\rho}\frac{\partial}{\partial\overline{\rho}}\left(\overline{\rho}\frac{\partial\tilde{\boldsymbol{\psi}}_{e,h}}{\partial\overline{\rho}}\right) + \left[(\boldsymbol{k}_{\rho e,h}\overline{\rho})^2 - m^2\boldsymbol{I}_{e,h}\right]\tilde{\boldsymbol{\psi}}_{e,h} = \mathbf{0}$$

with

$$\boldsymbol{T}_{e,h}^{-1}\underbrace{(\boldsymbol{\varepsilon}_{e,h} - \boldsymbol{P}_{e,h})}_{\boldsymbol{Q}_{e,h}}\boldsymbol{T}_{e,h} = \boldsymbol{k}_{\rho_{e,h}}^2$$

$\boldsymbol{\psi}_e$ and $\boldsymbol{\psi}_h$ are column vectors composed of the discretized potentials, $\overline{\rho} = k_o\rho$ is the normalized radial coordinate, m is the mode order in azimuthal direction and the matrixes $\boldsymbol{k}_{\rho e}$ and $\boldsymbol{k}_{\rho h}$ contain the propagation constant in the radial direction on the main diagonal. $\boldsymbol{T}_e$ and $\boldsymbol{T}_h$ are the transformation matrixes. $\boldsymbol{\varepsilon}_e$ and $\boldsymbol{\varepsilon}_h$ denote diagonal matrixes containing the values of the permittivity on the distinct line system. $\boldsymbol{P}_e$ and $\boldsymbol{P}_h$ are the difference operators for the second derivatives.

As a general solution of the uncoupled differential equations a linear combination of the Bessel function of the first and the second kind according to

$$\tilde{\boldsymbol{\psi}}_{e,h} = J_m(\boldsymbol{k}_{\rho_{e,h}}\overline{\rho})\mathbf{A}_{e,h} + Y_m(\boldsymbol{k}_{\rho_{e,h}}\overline{\rho})\mathbf{B}_{e,h}$$

can be used. This solution enables us to transform the quantities readily from one side of a layer to the other.

In accordance with the full vectorial character of the MoL we can evaluate all six field components from these potentials $\tilde{\boldsymbol{\psi}}_e$ and $\tilde{\boldsymbol{\psi}}_h$. Matching the tangential field components at the interfaces of the cylindrical rings yields an indirect eigenvalue problem. This eigenvalue problem is similar to that in [6], but now all matrixes are complex. This problem can be solved very efficiently by using the SVD (singular value decomposition) [12], [13]. One advantage of the SVD is that we obtain a real function with complex frequencies as argument instead of a complex function with a complex argument. On the other hand, the field distribution in the matching plane is provided directly by one of the transformation matrixes. For that very reason it is possible to decide what kind of mode was found.

3 Numerical Results

In literature an isolated dielectric resonator was used as a test example for three hybrid methods [2] [4] [5]. This resonator was investigated experimentally by [14], too. We also analyzed this isolated resonator, which has a radius of 5.25 mm, a height of 4.6 mm and a relative permittivity of 38. The resonant frequencies and the Q-factors due to radiation were determined using the described algorithm based on the MoL for the five lowest order modes. The results are illustrated in table 1 and table 2. Table 1 elucidates that our results for the values of the normalized resonant wavenumber ($k_o a$) are in good consistency with the results from the other numerical rigorous methods and the measurements.

Mode	$k_o a$ for resonance				
	MoL	Theory [2]	Theory [4]	Theory [5]	Measured [14]
$TE_{01\delta}$	0.537	0.531	0.534	0.535	0.533
$HEM_{11\delta}$	0.696	0.696	0.698	0.698	0.696
$HEM_{12\delta}$	0.732	0.730	0.731	0.731	0.726
$TM_{01\delta}$	0.827	0.827	0.829	0.827	0.824
$HEM_{21\delta}$	0.852	0.852	0.854	0.852	0.850

Table 1

Mode	Q-factor due to radiation				
	MoL	Theory [2]	Theory [4]	Theory [5]	Measured [14]
$TE_{01\delta}$	43.73	45.8	40.80	47	46.4
$HEM_{11\delta}$	31.46	30.7	30.85	31	30.3
$HEM_{12\delta}$	48.55	52.1	50.30	46	43.3
$TM_{01\delta}$	72.96	76.8	76.90	71	58.1
$HEM_{21\delta}$	344.29	327.1	337.66	324	346.1

Table 2

The variance between our results and the other results is always less than 1%. It can also be seen that, in the case of the Q-factor due to radiation in table 2, the highest deviation from our results to the others is 6%. The only exceptional case is that of the $TM_{01\delta}$ mode. Here all theoretical values are consistent whereas the value of this particular measurement differs extremly. As the authors of [14] explain, the reason lies in the fact that they had problems with interference between modes. In Fig. 3 a vector plot of

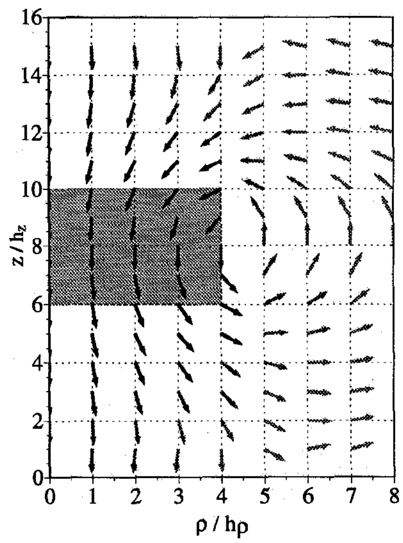

Fig. 3: Vector plot of the magnetic field profile

the magnetic field of the $TE_{01\delta}$ mode is illustrated. The gray area represents half the resonator structure at a height of 4.6mm and a radius of 5.25mm. The blackened intensity of the arrows is proportional to the value of the field magnitude. As expected, the electromagnetic behavior of the resonator is similar to that of a magnetic dipole.

4 Conclusion

A hybrid algorithm based on the MoL is presented for the analysis of multilayered resonator structures. Our model is compared with other models based on other methods and with experimental data for the example of an isolated dielectric resonator. The determination of the resonance frequencies and the quality factors shows that our results are in good consistency with the other results.

In addition to these first results we are going to present results for a dielectric conical resonator and a comparison to [10] at the conference. Furthermore it will be demonstrated that our model is well suited for the analysis of microstrip ring resonators.

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